Comparative Analysis of Pressure Reconstruction Methods from Simulation Impulse Data

https://doi.org/10.31713/MCIT.2025.068

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Abstract— **This thesis investigates the ill-posed problem of reconstructing the pressure time profile p(t) on a structure from discrete impulse data obtained in a 2D Discrete Element Method (DEM) simulation of blast loading. A systematic comparative analysis of six reconstruction methods is carried out: interval averaging, trapezoidal rule inversion, direct differentiation of cumulative impulse, cubic spline approximation, Savitzky-Golay filtering, and Tikhonov regularization with a non-negativity constraint. To assess the quality and physical adequacy of the solutions, a comprehensive system of metrics was developed, including smoothness, integral conservation (energy balance), and the fraction of non-physical negative values. For comparison with experimental data, a robust procedure is proposed that includes automatic time scale calibration and scale-independent signal shape validation. All results presented in the article are generated automatically within a single computational pipeline. The analysis shows that direct differentiation methods are critically unstable, while the Tikhonov regularization method with automatic parameter selection via Generalized Cross-Validation demonstrates the best trade-off between stability, smoothness, and fidelity to the source data.**

Keywords—pressure reconstruction, inverse problem, impulse, DEM, Tikhonov regularization, Savitzky-Golay filter, cubic splines

I. INTRODUCTION

The assessment of dynamic loads from shock waves on infrastructure elements is a critical task for ensuring their safety and stability. In many cases, both in physical experiments and computer simulations, direct measurement of pressure is difficult. Instead, the integral characteristic: impulse is easier to measure, by recording particle collisions with an obstacle in the simulation.

This work focuses on the inverse problem: restoring the time dependence of pressure $p(t)$ from a known time series of impulse increments $\Delta I(t)$, obtained from a 2D DEM (Discrete Element Method) simulation. From a mathematical point of view, this problem is ill-posed, as it requires differentiating noisy data, which is an unstable operation. Therefore, to obtain a physically adequate solution, it is necessary to apply regularization methods and consider a priori constraints, particularly the non-negativity of pressure $p(t) \geq 0$.

II. DEM SIMULATION AND IMPULSE MEASUREMENT

The data for the analysis is generated in a 2D DEM simulation, which models the movement of "air" particles colliding with a stationary modular structure [1]. A key aspect of the simulation is the use of a deterministic time axis based on the number of steps the physical engine processed during the simulation, which ensures full reproducibility of the results.

For each particle contact with the structure, the time of collision, mass, and velocity of the particle are recorded. The impulse of a single event is calculated as

$$\Delta I_{event} = |v| \cdot m.$$

III. COMPUTATIONAL PIPELINE

The data processing and pressure reconstruction are executed through an automated pipeline. This procedure commences with a preprocessing stage, wherein collision events are temporally sorted and aggregated within discrete time steps, enabling the calculation of precise time intervals (Δt). To address methods susceptible to outliers, a robust winsorization procedure is applied to the impulse data at the 1st and 99th percentiles, which mitigates the influence of extreme values without compromising the overall integral. Following preprocessing, time scale alignment is performed by automatically scaling the simulation's time axis to ensure accurate comparison with experimental data. The scaling factor is calibrated to match the full width at half maximum (FWHM) of a preliminary reconstruction with its experimental counterpart. Subsequently, pressure reconstruction is conducted using the six methodologies (M1-M6) detailed in Section 4, with careful consideration of non-uniform time steps (Δt). Reconstruction Methods (M1–M6)

Let us denote the discrete time intervals as Δt_k, the corresponding impulse increments as ΔI_k, and the cumulative impulse as $I_{cum}(t)$.

The initial methodology, Interval Averaging (M1), calculates pressure through the fundamental relation $p_k = \frac{\Delta I_k}{\Delta t_k}$. While this elemental method ensures integral conservation [2], it yields a piecewise-constant, non-smooth profile, thereby serving as a stable baseline for comparative analysis.

The second methodology, Trapezoidal Rule Inversion (M2), is based on the recursive formula $p_k = \frac{2 \cdot \Delta I_k}{\Delta t_k} - p_{k-1}$[3], initialized with $p_0 = 0$. In theory, this method should generate a linear pressure profile; however, in practice, it exhibits critical instability due to error propagation, which results in non-physical oscillations.

The third methodology, Differentiation of Cumulative Impulse (M3)[4], approximates pressure using the central difference formula:

$$p(t_k) \approx \frac{I_{k+1} - I_{k-1}}{t_{k+1} - t_{k-1}}.$$

This constitutes a direct numerical implementation of the definition of pressure as the impulse derivative. While it circumvents the recursive instability of M2, it is highly susceptible to the amplification of noise present in the source data.

The fourth methodology, Cubic Splines (M4)[5], involves approximating the cumulative impulse function $I_{cum}(t)$ with a smooth cubic spline, $S(t)$, followed by analytical differentiation to obtain the pressure profile: $p(t) = S'(t)$. This technique yields a continuously differentiable profile but may introduce numerical artifacts, such as oscillations and negative "tails," particularly at the interval boundaries.

The fifth methodology, the Savitzky-Golay Filter (M5), performs local polynomial smoothing on the cumulative impulse I_{cum} within a moving window [6], concurrently computing its derivative. The optimal parameters—a window size of 21 and a polynomial order of 2—were determined via a grid search methodology. This parameter set was selected for its efficacy in minimizing the trade-off between smoothness and the impulse balance error. This filter represents an effective technique for simultaneous smoothing and differentiation.

The final methodology, Tikhonov Regularization (M6) [7], is formulated as the solution to a system of linear equations $Ap = b$, where A is the integration operator matrix. The system is regularized to obtain a smooth and stable solution by minimizing the functional

$$|Ap - b|_2^2 + \lambda^2 |Dp|_2^2,$$

where D is the second-difference matrix (a "non-smoothness" operator), and λ is the regularization parameter.

IV. Conclusion

The data unequivocally demonstrates the critical instability of method M2, which is manifested in enormous roughness, a large fraction of negative values, and a catastrophic balance error. Method M3 is also unstable. Methods using smoothing (M4, M5, M6) show significantly better results. M5 provides the highest smoothness but has a small balance error and a fraction of negative values. Method M6 with GCV regularization and projection guarantees complete non-negativity with acceptable smoothness and a relatively small balance error, making it the most balanced.

V. References

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