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Combined Boundary Problem of 2D in the Geometrical Middle -1D in the Middle-1D Multi-Point type in the Non-Classical Treatment for 4D Bianchi Equation with Non-Smooth Coefficients, Arising in the Modeling of Vibration Processes and Integral Representation

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Abstract In this paper substantiated for a 4D Bianchi equation with non-smooth coefficients a four dimensional combined boundary problem - 4D combined boundary problem of 2D in the geometrical middle-1D in the middle-1D multi-point type with non-classical boundary conditions is considered, which requires no matching conditions. Equivalence of these conditions four dimensional boundary condition is substantiated classical, in the case if the solution of the problem in the isotropic S. L. Sobolev's space is found. The considered equation as a hyperbolic equation generalizes not only classic equations of mathematical physics (Laplace equation, telegraph equation, string vibration equation) and also many models differential equations (3D and 4D telegraph equation, 3D Bianchi equation, 3D and 4D wave equations and etc.). It is grounded that the 2D in the geometrical middle -1D in the middle -1D multi-point combined boundary conditions in the classic and non-classic treatment are equivalent to each other. Thus, namely in this paper, the non-classic problem with 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type conditions is grounded for a hyperbolic equation of fourth-order. For simplicity, this was demonstrated for one model case in one of S.L. Sobolev isotropic space $W_p^{(1,1,1,1)}(G)$.

Key words: 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type, 4D Bianchi equation, 4D mathematical modeling, hyperbolic equations, equation with non-smooth coefficients, equations with dominating mixed derivative.

2010 Mathematics Subject Classifications: 35L25, 35L35.

1. Introduction

Hyperbolic equations are attracted for sufficiently adequate description of a great deal of real processes occurring in the nature, engineering and etc. In particular, many processes arising in the theory of fluid filtration in cracked media are described by non-smooth coefficient hyperbolic equations.

Urgency of investigations conducted in this field is explained by appearance of local and non-local problems for non-smooth coefficients equations connected with different applied problems. Such type problems arise for example, while studying the problems of moisture, transfer in soils, heat transfer in heterogeneous media, diffusion of thermal neutrons in inhibitors, simulation of different biological processes, phenomena and etc. [1-3].

In the present paper, here consider four dimensional combined boundary problem- 2D in the geometrical middle -1D in the middle -1D multi-point combined boundary problem for 4D Bianchi equation with non-smooth coefficients. The coefficients in this hyperbolic equation are not necessarily differentiable; therefore, there does not exist a formally adjoint differential equation making a certain sense. For this reason, this question cannot be investigated by the well-known methods using classical integration by parts and Riemann functions or classical-type fundamental solutions. The theme of the present paper, devoted to the investigation 2D in the geometrical middle -1D in the middle -1D multi-point combined boundary problem for 4D Bianchi equations of hyperbolic type, according to the above-stated is very actual for the solution of theoretical and practical problems. From this point of view, the paper is devoted to the actual problems of applied mathematics and physics.

2. Problem Statement

Consider 4D Bianchi equation

$$(V_{1,1,1,1}u)(x,y,z,t) \equiv \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{m=0}^{1}\sum_{n=0}^{1} A_{i,j,m,n}(x,y,z,t) D_x^i D_y^j D_z^m D_t^n u(x,y,z,t) =$$

$$= f_{1,1,1,1}(x,y,z,t) \in L_p(G), \qquad (A_{1,1,1,1}(x,y,z,t) \equiv 1), \tag{1}$$

Here $u(x,y,z,t)$ is a desired function determined on G; $A_{i,j,m,n}(x,y,z,t)$ are the given measurable functions on $G = G_1 \times G_2 \times G_3 \times G_4$, where $G_1 = (x_0, h_1)$, $x_0 \ge 0$, $G_2 = (y_0, h_2)$, $G_3 = (z_0, h_3)$, $z_0 \ge 0$, $G_4 = (t_0, T)$; $f_{1,1,1,1}(x,y,z,t)$ is a given measurable function on G .

Equation (1) is a four dimensional Bianchi equation with four simple real characteristics $x = const$, $y = const$, $z = const$, $t = const$. Therefore, in some sense we can consider equation (1) as a hyperbolic equation. Equations of the form (1) are used in the modeling of vibration processes [4].

In the present paper 4D Bianchi equation (1) is considered in the general case when the coefficients $A_{i,j,m,n}(x,y,z,t)$ are non-smooth

functions satisfying only the following conditions:

$$A_{0,0,0,0}(x,y,z,t) \in L_p(G);\ A_{1,0,0,0}(x,y,z,t) \in L^{x,y,z,t}_{\infty,p,p,p}(G);\ A_{0,1,0,0}(x,y,z,t) \in L^{x,y,z,t}_{p,\infty,p,p}(G);$$

$$A_{0,0,1,0}(x,y,z,t) \in L^{x,y,z,t}_{p,p,\infty,p}(G);\ A_{0,0,0,1}(x,y,z,t) \in L^{x,y,z,t}_{p,p,p,\infty}(G);\ A_{1,1,0,0}(x,y,z,t) \in L^{x,y,z,t}_{\infty,\infty,p,p}(G);$$

$$A_{1,0,1,0}(x,y,z,t) \in L^{x,y,z,t}_{\infty,p,\infty,p}(G); \qquad A_{1,0,0,1}(x,y,z,t) \in L^{x,y,z,t}_{\infty,p,p,\infty}(G); \qquad A_{0,1,1,0}(x,y,z,t) \in L^{x,y,z,t}_{p,\infty,\infty,p}(G);$$

$$A_{0,1,0,1}(x,y,z,t) \in L^{x,y,z,t}_{p,\infty,p,\infty}(G); \qquad A_{0,0,1,1}(x,y,z,t) \in L^{x,y,z,t}_{p,p,\infty,\infty}(G); \qquad A_{1,1,1,0}(x,y,z,t) \in L^{x,y,z,t}_{\infty,\infty,\infty,p}(G);$$

$$A_{1,1,0,1}(x,y,z,t) \in L^{x,y,z,t}_{\infty,\infty,p,\infty}(G);\ A_{0,1,1,1}(x,y,z,t) \in L^{x,y,z,t}_{p,\infty,\infty,\infty}(G);\ A_{1,0,1,1}(x,y,z,t) \in L^{x,y,z,t}_{\infty,p,\infty,\infty}(G).$$

Under these conditions, we'll look for the solution $u(x,y,z,t)$ of equation (1)

in S.L.Sobolev isotropic space

$$W_p^{(1,1,1,1)}(G)\equiv\{u(x,y,z,t):D_x^iD_y^jD_z^mD_t^nu(x,y,z,t)\in L_p(G),\ \ i,j,m,n=\overline{0,1}\},$$

where $1\le p\le\infty$. $D_\xi^i=\partial^i/\partial\xi^i$ is a generalized differentiation operator in S.L.Sobolev sense, D_ξ^0 is an identity transformation operator. We'll define the norm in the space $W_p^{(1,1,1,1)}(G)$ by the equality

$$\|u\|_{W_p^{(1,1,1,1)}(G)}=\sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{m=0}^{1}\sum_{n=0}^{1}\left\|D_x^iD_y^jD_z^mD_t^nu\right\|_{L_p(G)}$$

For 4D Bianchi equation (1) we can give the classic form 2D in the geometrical middle -1D in the middle -1D multi-point combined boundary conditions in the form :

$$\begin{cases} u(x,y,z,t)\big|_{x=\sqrt{x_0h_1}}=B(y,z,t),(y,z,t)\in G_2\times G_3\times G_4;\\ u(x,y,z,t)\big|_{y=\frac{y_0+h_2}{2}}=P(x,z,t),(x,z,t)\in G_1\times G_3\times G_4;\\ u(x,y,z,t)\big|_{z=\sqrt{z_0h_3}}=\Psi(x,y,t),(x,y,t)\in G_1\times G_2\times G_4;\\ u(x,y,z,t)\big|_{t=t_k}=S_k(x,y,z),(x,y,z)\in G_1\times G_2\times G_3. \end{cases}\tag{2}$$

where $B(y,z,t)$, $P(x,z,t)$, $\Psi(x,y,t)$ and $S_k(x,y,z)$ are the given measurable functions on G. Notice that here we assume t_k, $k=1,2,...,N$ are the fixed points from $\bar{G}_4$. It is obvious that in the case of conditions (2), in addition to the conditions

$$B(y,z,t)\in W_p^{(1,1,1)}(G_2\times G_3\times G_4)\equiv\left\{\approx{B}(y,z,t):D_y^jD_z^mD_t^n\approx{B}(y,z,t)\in L_p(G_2\times G_3\times G_4),\ \ j,m,n=\overline{0,1}\right\};$$

$$P(x,z,t)\in W_p^{(1,1,1)}(G_1\times G_3\times G_4)\equiv\left\{\approx{P}(x,z,t):D_x^iD_z^mD_t^n\approx{P}(x,z,t)\in L_p(G_1\times G_3\times G_4),\ \ i,m,n=\overline{0,1}\right\};$$

$$\Psi(x,y,t)\in W_p^{(1,1,1)}(G_1\times G_2\times G_4)\equiv\left\{\approx{\Psi}(x,y,t):D_x^iD_y^jD_t^n\approx{\Psi}(x,y,t)\in L_p(G_1\times G_2\times G_4),\ \ i,j,n=\overline{0,1}\right\};$$

and

$$S_k(x,y,z)\in W_p^{(1,1,1)}(G_1\times G_2\times G_3)\equiv\left\{\approx{S}_k(x,y,z):D_x^iD_y^jD_z^m\approx{S}_k(x,y,z)\in L_p(G_1\times G_2\times G_3),\ \ i,j,m=\overline{0,1}\right\};$$

the given functions should also satisfy the following agreement conditions:

$$\begin{cases} B(y,z,t)\big|_{y=\frac{y_0+h_2}{2}}=P(x,z,t)\big|_{x=\sqrt{x_0h_1}},\\ B(y,z,t)\big|_{z=\sqrt{z_0h_3}}=\Psi(x,y,t)\big|_{x=\sqrt{x_0h_1}},\\ B(y,z,t)\big|_{t=t_k}=S_k(x,y,z)\big|_{x=\sqrt{x_0h_1}},\\ P(x,z,t)\big|_{z=\sqrt{z_0h_3}}=\Psi(x,y,t)\big|_{y=\frac{y_0+h_2}{2}},\\ P(x,z,t)\big|_{t=t_k}=S_k(x,y,z)\big|_{y=\frac{y_0+h_2}{2}},\\ \Psi(x,y,t)\big|_{t=t_k}=S_k(x,y,z)\big|_{z=\sqrt{z_0h_3}}. \end{cases}\tag{3}$$

Consider the following non-classical 2D in the geometrical middle -1D in the middle -1D multi-point combined boundary conditions :

$$
\begin{cases}
V^{(k)}_{0,0,0,0}u \equiv u(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, t_k) = f^{(k)}_{0,0,0,0} \in R; \\
(V^{(k)}_{1,0,0,0}u)(x) \equiv u_x(x, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, t_k) = f^{(k)}_{1,0,0,0}(x) \in L_p(G_1); \\
(V^{(k)}_{0,1,0,0}u)(y) \equiv u_y(\sqrt{x_0h_1}, y, \sqrt{z_0h_3}, t_k) = f^{(k)}_{0,1,0,0}(y) \in L_p(G_2); \\
(V^{(k)}_{0,0,1,0}u)(z) \equiv u_z(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, z, t_k) = f^{(k)}_{0,0,1,0}(z) \in L_p(G_3); \\
(V_{0,0,0,1}u)(t) \equiv u_t(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, t) = f_{0,0,0,1}(t) \in L_p(G_4); \\
(V^{(k)}_{1,1,0,0}u)(x,y) \equiv u_{xy}(x, y, \sqrt{z_0h_3}, t_k) = f^{(k)}_{1,1,0,0}(x,y) \in L_p(G_1 \times G_2); \\
(V^{(k)}_{1,0,1,0}u)(x,z) \equiv u_{xz}(x, \frac{y_0+h_2}{2}, z, t_k) = f^{(k)}_{1,0,1,0}(x,z) \in L_p(G_1 \times G_3); \\
(V_{1,0,0,1}u)(x,t) \equiv u_{xt}(x, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, t) = f_{1,0,0,1}(x,t) \in L_p(G_1 \times G_4); \\
(V^{(k)}_{0,1,1,0}u)(y,z) \equiv u_{yz}(\sqrt{x_0h_1}, y, z, t_k) = f^{(k)}_{0,1,1,0}(y,z) \in L_p(G_2 \times G_3); \\
(V_{0,1,0,1}u)(y,t) \equiv u_{yt}(\sqrt{x_0h_1}, y, \sqrt{z_0h_3}, t) = f_{0,1,0,1}(y,t) \in L_p(G_2 \times G_4); \\
(V_{0,0,1,1}u)(z,t) \equiv u_{zt}(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, z, t) = f_{0,0,1,1}(z,t) \in L_p(G_3 \times G_4); \\
(V^{(k)}_{1,1,1,0}u)(x,y,z) \equiv u_{xyz}(x, y, z, t_k) = f^{(k)}_{1,1,1,0}(x,y,z) \in L_p(G_1 \times G_2 \times G_3); \\
(V_{1,1,0,1}u)(x,y,t) \equiv u_{xyt}(x, y, \sqrt{z_0h_3}, t) = f_{1,1,0,1}(x,y,t) \in L_p(G_1 \times G_2 \times G_4); \\
(V_{0,1,1,1}u)(y,z,t) \equiv u_{yzt}(\sqrt{x_0h_1}, y, z, t) = f_{0,1,1,1}(y,z,t) \in L_p(G_2 \times G_3 \times G_4); \\
(V_{1,0,1,1}u)(x,z,t) \equiv u_{xzt}(x, \frac{y_0+h_2}{2}, z, t) = f_{1,0,1,1}(x,z,t) \in L_p(G_1 \times G_3 \times G_4).
\end{cases}
\tag{4}
$$

3.

Methodology

Therewith, the important principal moment is that the considered equation possesses nonsmooth coefficients satisfying only some p -integrability and boundedness conditions i.e. the considered hyperbolic operator $V_{1,1,1,1}$ has no traditional conjugated operator. In other words, the Riemann function for this equation can't be investigated by the classical method of characteristics. In the papers [5-7] the Riemann function is determined as the solution of an integral equation. This is more natural than the classical way for deriving the Riemann function. The matter is that in the classic variant, for determining the Riemann function, the rigid smooth conditions on the coefficients of the equation are required.

The Riemann's method does not work for hyperbolic equations with non-smooth coefficients. Especially it should be noted that a variety of boundary-value problems for the equations of Bianchi studied in [8-13] and etc.

In the present paper, a method that essentially uses modern methods of the theory of functions and functional analysis is worked out for investigations of such problems. In the main, this method it requested in conformity to hyperbolic equations of fourth-order with simple real characteristics. Notice that, in this paper the considered equation is a generation of many model equations of some processes (for example, 3D and 4D telegraph equation, 3D Bianchi equation, 3D and 4D wave equations and etc).

If the function $u \in W_p^{(1,1,1,1)}(G)$ is a solution of the classical form 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type (1), (2), then it is also a solution of problem (1), (4) for $f_{i_1,i_2,i_3,i_4}^{(k)}$ *and* f_{i_1,i_2,i_3,i_4}, $i_k = \overline{0,1}, k = \overline{1,4}$ defined by the following e qualities:

$$f_{0,0,0,0}^{(k)} = B(\frac{y_0 + h_2}{2}, \sqrt{z_0 h_3}, t_k) = P(\sqrt{x_0 h_1}, \sqrt{z_0 h_3}, t_k) = \Psi(\sqrt{x_0 h_1}, \frac{y_0 + h_2}{2}, t_k) = S_k(\sqrt{x_0 h_1}, \frac{y_0 + h_2}{2}, \sqrt{z_0 h_3});$$

$$f_{1,0,0,0}^{(k)}(x) = \frac{\partial P(x,z,t)}{\partial x} /_{z=\sqrt{z_0 h_3}, t=t_k} = \frac{\partial \Psi(x,y,t)}{\partial x} /_{y=\frac{y_0+h_2}{2}, t=t_k} = \frac{\partial S_k(x,y,z)}{\partial x} /_{y=\frac{y_0+h_2}{2}, z=\sqrt{z_0 h_3}};$$

$$f_{0,1,0,0}^{(k)}(y) = \frac{\partial \Psi(x,y,t)}{\partial y} /_{x=\sqrt{x_0 h_1}, t=t_k} = \frac{\partial S_k(x,y,z)}{\partial y} /_{x=\sqrt{x_0 h_1}, z=\sqrt{z_0 h_3}} = \frac{\partial B(y,z,t)}{\partial y} /_{z=\sqrt{z_0 h_3}, t=t_k};$$

$$f_{0,0,1,0}^{(k)}(z) = \frac{\partial B(y,z,t)}{\partial z} /_{y=\frac{y_0+h_2}{2}, t=t_k} = \frac{\partial S_k(x,y,z)}{\partial z} /_{x=\sqrt{x_0 h_1}, y=\frac{y_0+h_2}{2}} = \frac{\partial P(x,z,t)}{\partial z} /_{x=\sqrt{x_0 h_1}, t=t_k};$$

$$f_{0,0,0,1}(t) = \frac{\partial B(y,z,t)}{\partial t} /_{y=\frac{y_0+h_2}{2}, z=\sqrt{z_0 h_3}} = \frac{\partial P(x,z,t)}{\partial t} /_{x=\sqrt{x_0 h_1}, z=\sqrt{z_0 h_3}} = \frac{\partial \Psi(x,y,t)}{\partial t} /_{x=\sqrt{x_0 h_1}, y=\frac{y_0+h_2}{2}};$$

$$f_{1,1,0,0}^{(k)}(x,y) = \frac{\partial^2 S_k(x,y,z)}{\partial x \partial y} /_{z=\sqrt{z_0 h_3}} = \frac{\partial^2 \Psi(x,y,t)}{\partial x \partial y} /_{t=t_k};$$

$$f_{1,0,1,0}^{(k)}(x,z) = \frac{\partial^2 S_k(x,y,z)}{\partial x \partial z} /_{y=\frac{y_0+h_2}{2}} = \frac{\partial^2 P(x,z,t)}{\partial x \partial z} /_{t=t_k};$$

$$f_{1,0,0,1}(x,t) = \frac{\partial^2 P(x,z,t)}{\partial x \partial t} /_{z=\sqrt{z_0 h_3}} = \frac{\partial^2 \Psi(x,y,t)}{\partial x \partial t} /_{y=\frac{y_0+h_2}{2}};$$

$$f_{0,1,1,0}^{(k)}(y,z) = \frac{\partial^2 B(y,z,t)}{\partial y \partial z} /_{t=t_k} = \frac{\partial^2 S_k(x,y,z)}{\partial y \partial z} /_{x=\sqrt{x_0 h_1}};$$

$$f_{0,1,0,1}(y,t) = \frac{\partial^2 B(y,z,t)}{\partial y \partial t} /_{z=\sqrt{z_0 h_3}} = \frac{\partial^2 \Psi(x,y,t)}{\partial y \partial t} /_{x=\sqrt{x_0 h_1}};$$

$$f_{0,0,1,1}(z,t) = \frac{\partial^2 B(y,z,t)}{\partial z \partial t} /_{y=\frac{y_0+h_2}{2}} = \frac{\partial^2 P(x,z,t)}{\partial z \partial t} /_{x=\sqrt{x_0 h_1}};$$

$$f_{1,1,1,0}^{(k)}(x,y,z) = \frac{\partial^3 S_k(x,y,z)}{\partial x \partial y \partial z}; \quad f_{1,1,0,1}(x,y,t) = \frac{\partial^3 \Psi(x,y,t)}{\partial x \partial y \partial t};$$

$$f_{0,1,1,1}(y,z,t) = \frac{\partial^3 B(y,z,t)}{\partial y \partial z \partial t}; \quad f_{1,0,1,1}(x,z,t) = \frac{\partial^3 P(x,z,t)}{\partial x \partial z \partial t};$$

The inverse one is easily proved. In other words, if the function $u \in W_p^{(1,1,1,1)}(G)$ is a solution of problem (1), (4), then it is also a solution of problem (1), (2) for the following functions:

$$B(y,z,t)=f^{(k)}_{0,0,0,0}+\int\limits_{\frac{y_0+h_2}{2}}^{y}f^{(k)}_{0,1,0,0}(\eta)d\eta+\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{0,0,1,0}(\gamma)d\gamma+$$
$$+\int\limits_{t_k}^{t}f_{0,0,0,1}(\tau)d\tau+\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{0,1,1,0}(\eta,\gamma)d\eta d\gamma+$$
$$+\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{t_k}^{t}f_{0,1,0,1}(\eta,\tau)d\eta d\tau+\int\limits_{\sqrt{z_0h_3}}^{z}\int\limits_{t_k}^{t}f_{0,0,1,1}(\gamma,\tau)d\gamma d\tau+ \tag{5}$$
$$+\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{\sqrt{z_0h_3}}^{z}\int\limits_{t_k}^{t}f_{0,1,1,1}(\eta,\gamma,\tau)d\eta d\gamma d\tau;$$

$$P(x,z,t)=f^{(k)}_{0,0,0,0}+\int\limits_{\sqrt{x_0h_1}}^{x}f^{(k)}_{1,0,0,0}(\xi)d\xi+\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{0,0,1,0}(\gamma)d\gamma+$$
$$+\int\limits_{t_k}^{t}f_{0,0,0,1}(\tau)d\tau+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{1,0,1,0}(\xi,\gamma)d\xi d\gamma+ \tag{6}$$
$$+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{t_k}^{t}f_{1,0,0,1}(\xi,\tau)d\xi d\tau+\int\limits_{\sqrt{z_0h_3}}^{z}\int\limits_{t_k}^{t}f_{0,0,1,1}(\gamma,\tau)d\gamma d\tau+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\sqrt{z_0h_3}}^{z}\int\limits_{t_k}^{t}f_{1,0,1,1}(\xi,\gamma,\tau)d\xi d\gamma d\tau;$$

$$\Psi(x,y,t)=f^{(k)}_{0,0,0,0}+\int\limits_{\sqrt{x_0h_1}}^{x}f^{(k)}_{1,0,0,0}(\xi)d\xi+\int\limits_{\frac{y_0+h_2}{2}}^{y}f^{(k)}_{0,1,0,0}(\eta)d\eta+$$
$$+\int\limits_{t_k}^{t}f_{0,0,0,1}(\tau)d\tau+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\frac{y_0+h_2}{2}}^{y}f^{(k)}_{1,1,0,0}(\xi,\eta)d\xi d\eta+ \tag{7}$$
$$+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{t_k}^{t}f_{1,0,0,1}(\xi,\tau)d\xi d\tau+\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{t_k}^{t}f_{0,1,0,1}(\eta,\tau)d\eta d\tau+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{t_k}^{t}f_{1,1,0,1}(\xi,\eta,\tau)d\xi d\eta d\tau;$$

$$S_k(x,y,z)=f^{(k)}_{0,0,0,0}+\int\limits_{\sqrt{x_0h_1}}^{x}f^{(k)}_{1,0,0,0}(\xi)d\xi+\int\limits_{\frac{y_0+h_2}{2}}^{y}f^{(k)}_{0,1,0,0}(\eta)d\eta+\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{0,0,1,0}(\gamma)d\gamma+$$
$$+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\frac{y_0+h_2}{2}}^{y}f^{(k)}_{1,1,0,0}(\xi,\eta)d\xi d\eta+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{1,0,1,0}(\xi,\gamma)d\xi d\gamma+\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{0,1,1,0}(\eta,\gamma)d\eta d\gamma+ \tag{8}$$
$$+\int\limits_{\sqrt{x_0h_1}}^{x}\int\limits_{\frac{y_0+h_2}{2}}^{y}\int\limits_{\sqrt{z_0h_3}}^{z}f^{(k)}_{1,1,1,0}(\xi,\eta,\gamma)d\xi d\eta d\gamma.$$

Note that the functions (5)-(8) possess one important property, more exactly, for all $f^{(k)}_{i_1,i_2,i_3,i_4}$ *and* f_{i_1,i_2,i_3,i_4}, the agreement conditions (3) possessing the above-mentioned properties are fulfilled for them automatically. Therefore, equalities (5)-(8) may be considered as a general kind of all the functions

$B(y,z,t)$, $P(x,z,t)$, $\Psi(x,y,t)$ and $S_k(x,y,z)$ satisfying the agreement conditions (3).

We have thereby proved the following assertion.

Theorem . The 4D combined boundary problems of 2D in the geometrical middle -1D in the middle -1D multi-point type of the form (1), (2) and the non-classical form (1), (4) are equivalent.

Note that the 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type in the non-classical treatment (1), (4) can be studied with the use of multi-point integral representations of special form for the functions $u \in W_p^{(1,1,1,1)}(G)$ [14-21],

$$
\begin{aligned}
u(x,y,z,t) &= u(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, t_k) + \int_{\sqrt{x_0h_1}}^{x} u_\xi(\xi, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, t_k)d\xi + \\
&+ \int_{\frac{y_0+h_2}{2}}^{y} u_\eta(\sqrt{x_0h_1}, \eta, \sqrt{z_0h_3}, t_k)d\eta + \int_{\sqrt{z_0h_3}}^{z} u_\gamma(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, \gamma, t_k)d\gamma + \\
&+ \int_{t_k}^{t} u_\tau(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, \tau)d\tau + \int_{\sqrt{x_0h_1}}^{x}\int_{\frac{y_0+h_2}{2}}^{y} u_{\xi\eta}(\xi, \eta, \sqrt{z_0h_3}, t_k)d\xi d\eta + \\
&+ \int_{\sqrt{x_0h_1}}^{x}\int_{\sqrt{z_0h_3}}^{z} u_{\xi\gamma}(\xi, \frac{y_0+h_2}{2}, \gamma, t_k)d\xi d\gamma + \int_{\sqrt{x_0h_1}}^{x}\int_{t_k}^{t} u_{\xi\tau}(\xi, \frac{y_0+h_2}{2}, \sqrt{z_0h_3}, \tau)d\xi d\tau + \\
&+ \int_{\frac{y_0+h_2}{2}}^{y}\int_{\sqrt{z_0h_3}}^{z} u_{\eta\gamma}(\sqrt{x_0h_1}, \eta, \gamma, t_k)d\eta d\gamma + \int_{\frac{y_0+h_2}{2}}^{y}\int_{t_k}^{t} u_{\eta\tau}(\sqrt{x_0h_1}, \eta, \sqrt{z_0h_3}, \tau)d\eta d\tau + \\
&+ \int_{\sqrt{z_0h_3}}^{z}\int_{t_k}^{t} u_{\gamma\tau}(\sqrt{x_0h_1}, \frac{y_0+h_2}{2}, \gamma, \tau)d\gamma d\tau + \int_{\sqrt{x_0h_1}}^{x}\int_{\frac{y_0+h_2}{2}}^{y}\int_{\sqrt{z_0h_3}}^{z} u_{\xi\eta\gamma}(\xi, \eta, \gamma, t_k)d\xi d\eta d\gamma + \\
&+ \int_{\sqrt{x_0h_1}}^{x}\int_{\frac{y_0+h_2}{2}}^{y}\int_{t_k}^{t} u_{\xi\eta\tau}(\xi, \eta, \sqrt{z_0h_3}, \tau)d\xi d\eta d\tau + \int_{\frac{y_0+h_2}{2}}^{y}\int_{\sqrt{z_0h_3}}^{z}\int_{t_k}^{t} u_{\eta\gamma\tau}(\sqrt{x_0h_1}, \eta, \gamma, \tau)d\eta d\gamma d\tau + \\
&+ \int_{\sqrt{x_0h_1}}^{x}\int_{\sqrt{z_0h_3}}^{z}\int_{t_k}^{t} u_{\xi\gamma\tau}(\xi, \frac{y_0+h_2}{2}, \gamma, \tau)d\xi d\gamma d\tau + \int_{\sqrt{x_0h_1}}^{x}\int_{\frac{y_0+h_2}{2}}^{y}\int_{\sqrt{z_0h_3}}^{z}\int_{t_k}^{t} u_{\xi\eta\gamma\tau}(\xi, \eta, \gamma, \tau)d\xi d\eta d\gamma d\tau.
\end{aligned}
$$

4. Result

So, the classical form 4D combined boundary problems of 2D in the geometrical middle -1D in the middle -1D multi-point type (1), (2) and in non-classical treatment (1), (4) are equivalent in the general case. However, the 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type in non-classical statement (1), (4) is more natural by statement than problem (1), (2). This is connected with the fact that in statement of problem (1), (4) the right sides of boundary conditions don't require additional conditions of agreement type. Note that some boundary-value problems in non-classical treatments for hyperbolic and also pseudoparabolic equations and their applications to optimal control theory were investigated in the author's papers [22-35].

5. Discussion and Conclusions

In this paper a non-classical type 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type is substantiated for a 4D Bianchi equation with non-smooth coefficients and with a fourth-order dominating derivative. Classic 4D combined boundary of 2D in the geometrical middle -1D in the middle -1D multi-point type conditions are reduced to non-classic 4D combined boundary of 2D in the geometrical middle -1D in the middle -1D multi-point type conditions by means of integral representations. Such statement of the problem has several advantages:

1) No additional agreement conditions are required in this statement;

2) One can consider this statement as a 4D combined boundary problem of 2D in the geometrical middle -1D in the middle -1D multi-point type formulated in terms of traces in the S.L. Sobolev isotropic space $W_p^{(1,1,1,1)}(G)$;

3) In this statement the considered 4D Bianchi equation is a generalization of many model differential equations of some processes (e.g. 3D and 4D telegraph equation, 3D Bianchi equation, 3D and 4D wave equations and etc.).

REFERENCES

[1] Mamedov, I.G., Abdullayeva, A.J. Integral representation of solution of middle boundary problem for one integro-differential equation 3D Bianchi with non-smooth coefficients // "Operators, Functions, and Systems of Mathematical Physics" conference, -Baku: Khazar University, -10-14 June, -2019, -p.71-72.

[2] Məmmədov, İ.Q., Abdullayeva, A.C. Bir anizotrop Sobolev fəzasında funksiyaların inteqral göstərilişi və onun tətbiqi barədə // "Riyaziyyatın fundamental problemləri və intellektual texnologiyaların təhsildə tətbiqi" II Respublika elmi konfransı, -Sumqayıt: -15-16 dekabr -2022, -s.57-58.

[3] Abdullayeva, A.J. Fundamental solution of the boundary value problem for a hyperbolic integro-differential equation of the 3D Bianchi type // -Baku: Proceedings of the Institute of Applied Mathematics, -2022. v.11, issue 2, -p.159-190,

[4] Mamedov, I.G., Abdullayeva, A.J. One 3D in the Geometrical Middle Problem in the Non-classical Treatment for one 3D Bianchi Integro-differential Equation with Non-smooth Coefficients // -Baku: Caspian Journal of Applied Mathematics, -2018. v. 6, -pp. 73-81.

[5] Məmmədov, İ.Q., Abdullayeva, A.C. Azərbaycanda hərbi sənaye kompleksinin və hərbi hissələrdə tikilən binaların seysmik dayanıqlıq məsələlərinin riyazi modelləşdirilməsi barədə // Milli Müdafiə Universitetinin Hərbi Elmi Tədqiqat İnstitutu, Müasir Radotexniki Silahlar Respublika elmi-praktik konfransının materialları , -12 13 mart, -2024, -s.256-259.