MCIT Modeling, control & information technologies

Monitoring of rolling bearing with inner race fault using parameters of vibration birhythmic stochastic recurrence

https://doi.org/10.31713/MCIT.2025.076

Roman Pelypets
Lviv Polytechnic National University, Lviv, Ukraine
pelypets.ri@gmial.com

Roman Yuzefovych
Lviv Polytechnic National University, Lviv, Ukraine
Karpenko Physico-mechanical Institute
of NAS of Ukraine, Lviv, Ukraine
roman.yuzefovych@gmail.com

Ihor Javorskyj
Karpenko Physico-mechanical Institute
of NAS of Ukraine, Lviv, Ukraine
University of Science and Technology,
Bydgoszcz, Poland
javor@pbs.edu.pl

Oleh Lychak
Karpenko Physico-mechanical Institute
of NAS of Ukraine, Lviv, Ukraine
olehlychak2003@yahoo.com

Roman Sliepko
Karpenko Physico-mechanical Institute of NAS of Ukraine, Lviv, Ukraine
roman.slyepko@gmail.com

Abstract— **An analysis of the covariance and spectral structure of the Hilbert transform of biperiodically non-stationary random processes, which are a model of signals with double rhythmicity, are provided. The obtained relations for the cross-covariance and cross-spectral characteristics of the signal and its Hilbert transform.**

Keywords — bi-periodically non-stationary random process, Hilbert transform, covariance and spectral components, vibration, hidden periodicities.

I. Introduction

The analysis of vibrations origin from the damaged rotating machines on the basis of their model in the form of periodically non-stationary random processes (PNRP) enables to detect the mechanism faults on early stages of their development as well as to determine the parameters which characterize faults [1, 3, 4]. The powers of the resonance oscillations increase when the shaft passes through a fault zone. As the result of recurrent passings, the vibration signal acquires the properties of the birhythmical recurrence. The shaft rotation frequency and ball pass frequency on internal ring (BPFI) can be considered as the signal basic frequencies. In the present paper, we prove that on the basis signal model, in the form of the biperiodically non-stationary processes (BPNRP), we can discover a new its regularities, which describe the properties of the joint modulation deterministic and stochastic components for two recurrences. It is also to substantiate the new bearing condition indicators for its monitoring and the improvement of efficiency for the fault detection.

II. Covariance and Spectral Properties of BPNRP.

The mean function of a BPNRP $m_\xi(t)=E\xi(t)$ and its covariance function $b_\xi(t,u)=E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+u)$, $\overset{\circ}{\xi}(t)=\xi(t)-m_\xi(t)$, where E is the mathematical expectation operator, are biperiodic functions of time and can be represented by Fourier series:

$$m_\xi(t)=\sum_{k,l\in Z} m_{kl}^{(\xi)} e^{i\omega_{kl}t} = m_{00}^{(\xi)}+\sum_{k=0}^{\infty}\sum_{l=1}^{\infty}\left(m_{kl}^{c}\cos\omega_{kl}t+m_{kl}^{s}\sin\omega_{kl}t\right)+ +\sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\left(m_{k,-l}^{c}\cos\omega_{k,-l}t+m_{k,-l}^{s}\sin\omega_{k,-l}t\right), \quad (1)$$

$$b_\xi(t,u)=\sum_{k,l\in Z} B_{kl}^{(\xi)}(u) e^{i\omega_{kl}t} = B_{00}^{(\xi)}(u)+\sum_{k=0}^{\infty}\sum_{l=1}^{\infty}\left[B_{kl}^{c}(u)\cos\omega_{kl}t+B_{kl}^{s}(u)\sin\omega_{kl}t\right]+ +\sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\left[B_{k,-l}^{c}(u)\cos\omega_{k,-l}t+B_{k,-l}^{s}(u)\sin\omega_{k,-l}t\right]. \quad (2)$$

Here Z is the set of integers, $m_{kl}^{(\xi)} = \frac{1}{2}\left(m_{kl}^{c} - i m_{kl}^{s}\right)$ and

$$B_{kl}^{(\xi)}(u) = \frac{1}{2}\left[B_{kl}^{c}(u) - iB_{kl}^{s}(u)\right],$$

$m_{-k,-l}^{(\xi)} = \overline{m}_{kl}^{(\xi)}$, $B_{-k,-l}^{(\xi)}(u) = \overline{B}_{kl}^{(\xi)}(u)$, where "¯" – denotes conjugation, $\omega_{kl} = k\frac{2\pi}{P_1} + l\frac{2\pi}{P_2}$, P_1 and P_2 are periods. The process $\xi(t)$ can be represented in the form of a stochastic series:

$$\xi(t) = \sum_{k,l \in Z} \xi_{kl}(t) e^{i w_{kl} t}, \qquad (3)$$

where $\xi_{kl}(t) = \frac{1}{2}\left[\xi_{kl}^{c}(t) - \xi_{kl}^{s}(t)\right]$, $\xi_{-k,-l}(t) = \overline{\xi}_{kl}(t)$, are jointly stationary random processes.

III. HILBERT TRANSFORM

Consider that the random process $\xi(t)$ (3) has a zeroth constant component $m_{00} = 0$. Then, there exists the Hilbert transform

$$\eta(t) = H\{\xi(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\xi(\tau)}{t-\tau} d\tau, \qquad (4)$$

and for its mathematical expectation, we have:

$$m_\eta(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m_\xi(t)}{t-\tau} d\tau.$$

After substituting into this formula series (1), we obtain:

$$m_\eta(t) = \sum_{k=0}^{\infty}\sum_{l=1}^{\infty}\left(m_{kl}^{c}\sin\omega_{kl}t - m_{kl}^{s}\cos\omega_{kl}t\right) + \\ + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\left(m_{k,-l}^{c}\sin\omega_{k,-l}t - m_{k,-l}^{s}\cos\omega_{k,-l}t\right).$$

Here it is taken into account that the Hilbert transform shifts the phases of the harmonics of the mathematical expectation (1) by $-\frac{\pi}{2}$. The mathematical expectation of the analytic signal $\zeta(t) = \xi(t) + i\eta(t)$ then has the form:

$$m_\zeta(t) = 2\left[\sum_{k=0}^{\infty}\sum_{l=1}^{\infty} m_{kl} e^{i\omega_{kl}t} + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty} m_{k,-l} e^{i\omega_{k,-l}t}\right].$$

A BPNRP $\xi(t)$, whose covariance function is represented by the series (2), and its Hilbert transform (4) are jointly biperiodically non-stationary processes, and their auto- and cross-covariance components are related by the ratios:

$$B_{kl}^{(\xi\eta)}(u) = \int_{-\infty}^{\infty} h(u-\tau) B_{kl}^{(\xi)}(\tau) d\tau,$$

$$B_{kl}^{(\eta\xi)}(u) = -\int_{-\infty}^{\infty} h(u-\tau) B_{kl}^{(\eta)}(\tau) d\tau,$$

$$B_{kl}^{(\xi)}(u) = -\int_{-\infty}^{\infty} h(u-\tau) B_{kl}^{(\xi\eta)}(\tau) d\tau,$$

$$B_{kl}^{(\eta)}(u) = \int_{-\infty}^{\infty} h(u-\tau) B_{kl}^{(\eta\xi)}(\tau) d\tau,$$

where $h(\tau) = (\pi\tau)^{-1}$ is the impulse response of the Hilbert transform, which means that the covariance components $B_{kl}^{(\xi)}(u)$ and $B_{kl}^{(\xi\eta)}(u)$, as well as $B_{kl}^{(\eta\xi)}(u)$ and $B_{kl}^{(\eta)}(u)$, are Hilbert pairs.

The zeroth covariance component of a BPNRP signal and its Hilbert transform are equal, and their zeroth cross-covariance components differ only by sign, they are odd functions and are determined by the formula:

$$B_{00}^{(\xi\eta)}(u) = 2\int_{0}^{\infty} f_{00}^{(\xi)}(\omega)\sin\omega u \, d\omega.$$

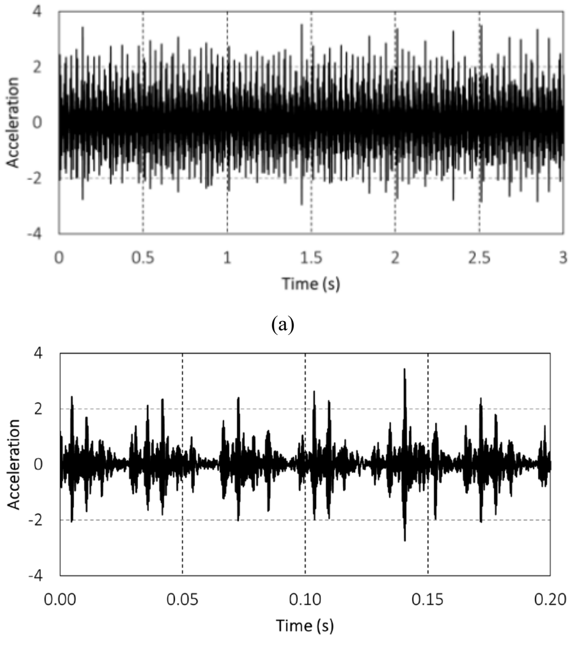
The cross-covariance function of a BPNRP signal and its Hilbert transform, as well as the autocovariance function of the latter, vary bi-periodically with time, and their Fourier coefficients are determined by the formulas:

$$B_{kl}^{(\eta)}(u) = \int_{-\infty}^{0} f_{kl}^{(\xi)}(\omega) e^{i\omega u} d\omega - \int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega) e^{i\omega u} d\omega \\ + \int_{\omega_{kl}}^{\infty} f_{kl}^{(\xi)}(\omega) e^{i\omega u} d\omega,$$

$$B_{kl}^{(\xi\eta)}(u) = i\int_{0}^{\infty}\left[f_{kl}^{(\xi)}(\omega + \omega_{kl}) e^{-i\omega u} - f_{kl}^{(\xi)}(\omega) e^{i\omega u}\right] d\omega.$$

IV. TIME SERIES BPNRP ANALYSIS

The segment of the recorded vibration signal realization from bearing with fault is presented in Fig. 1. The width of the inner race fault is 0.56 mm.

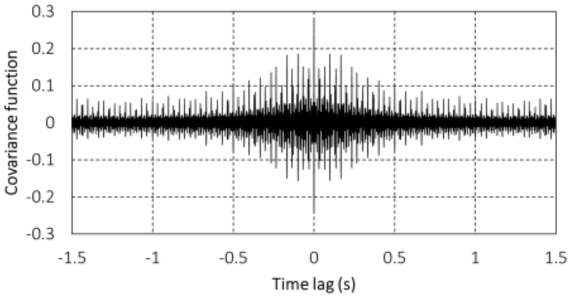


(a)

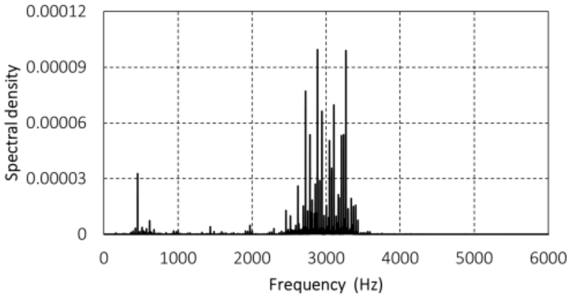
(b)

Fig. 1. Segment of vibration realization for diferent scales (a,b)

It can be seen in Fig. 2 that the vibrations have a form of the oscillatory groups following one after other with the time intervals that are close to shaft rotation period. The covariance function has the form undamped oscillatory groups. For time lag values outside 1.5s this graph turns into undamped oscillations. The spectral density estimator has the comb-form with different peak heights.



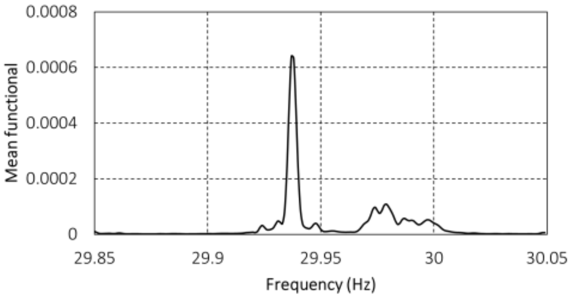
(a)



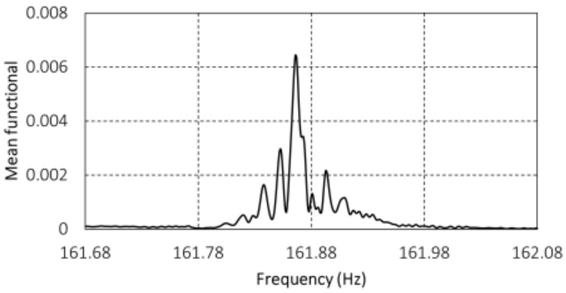
(b)

Fig. 2. Estimator of raw signal covariance function(a) and spectral density(b)

As it can be seen in Fig. 3, the graph has clear-cut maximum at point 29.937 Hz, which we take as the estimator of the low basic frequency and at point 161.862 Hz, which we take as the high frequency. [1]



(a)



(b)

Fig. 3. Dependence of variance functionals on test frequencies(a, b)

Based on the found basic frequencies we can calculate the following combination harmonics in Table 1.

Table 1. Frequencies of mean combination harmonics (Hz)

l \ k	3	4	...	17	18	19	20
-1	455.649	617.511	...	2721.717	2883.579	3045.441	3207.303
0	485.586	647.448	...	2751.654	2913.515	3075.378	3237.240
1	515.523	677.385	...	2781.591	2943.453	3105.315	3267.177

After extraction combination harmonics we can calculate the mean function of raw signal.

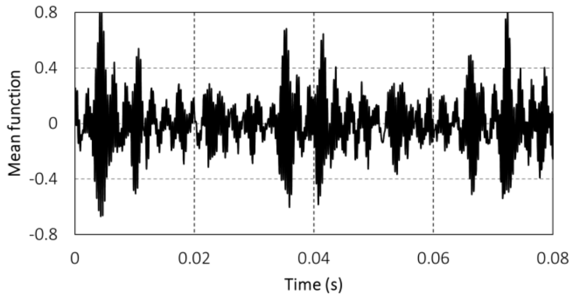


Fig. 4. Mean function of raw signal

The groups which follow one after other with intervals that are close to the shaft rotation period are mostly clearly observed in Fig. 4.

V. Conclusion

It is shown that the BPNRP is the adequate model for the vibrations of rolling bearing with inner race fault. Using quasi-LS techniques enables to estimate BPNRP basic frequencies (shaft rotation frequency and BPFI). Signal spectra consist of the periodical components with two basic frequencies and their multiples, and also the combination component. It is displayed the variance amplitude spectra of the signal and its Hilbert transform are identical, so the common used in literature definition of the squared envelope as the squared modulus of analysis signal is not correct in this case.

References

[1] Javorskyj I.M. Mathematical models and analysis of stochastic oscillations – Lviv, IPM NAS of Ukraine, 2013. – 802 P. (in Ukrainian)

[2] Napolitano A. Cyclostationary processes and time Series: Theory, applications, and generalizations Academic Press, Elsevier, London (2020).

[3] Javorskyj I., Yuzefovych R., Kravets I., Matsko I. Methods of periodically correlated random processes and their generalizations. In: Cyclostationarity: Theory and Methods. Lecture Notes in Mechanical Engineering. New York:Springer (2014), 73-93.

[4] Javorskyj I., Yuzefovych R., Kravets I., Matsko I. Periodically correlated processes: applications in early diagnostics of mechanical systems. Mechanical Systems and Signal Processing (2017), 83:406-438.

[5] Javorskyj I., Yuzefovych R., Lychak O., Semenov P., Shyepko R. Detection of distributed and localized faults in rotating machines using periodically non-stationary covariance analysis of vibrations. Measurement Science and Technology (2023), 34:065-102

[6] Javorskyj I., Yuzefovych R., Lychak O., Trokhym G., Varyvoda M. Methods of periodically non-stationary random processes for vibrations monitoring of rolling bearing with damaged outer race. Digital Signal Processing (2024), 145:104343.