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Modeling of Two-phase Filtration Processes and Homogenization

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Abstract—**Unsteady processes of two-phase flow filtrations for immiscible liquids in porous media with a periodic structure are considered. The Masket-Leveret model is adopted as a mathematical model for describing such processes for media formed by a large number of blocks with low permeability and separated by a connected system of faults with high permeability. This model is typical for modeling the processes of gas production, oil production and diffusion of pollutants (for example, oil products) in groundwater. The assumption of medium periodicity is a model as simply described, since it is not known how to describe real porous media. However, sometimes the results for periodic media can be generalized to stochastic media. In addition, complex media can sometimes be approximated by periodic or quasi-periodic, which are also reduced to periodic.**

Keywords— homogenized problems; porous media; approximating solutions; initial boundary value problems.

I. Introduction

The problem of modeling nonstationary processes of mutual filtration of multiphase flows of immiscible fluids in porous media arises in the development of oil and gas fields, the search for methods to prevent man-made pollution (e.g., petroleum products) in groundwater, and the search for ways to purify such waters. Direct practical study of such processes using technical observation methods is practically impossible, as it is necessary to install a large number of sensors over large areas and at various depths to understand the dynamics of the combined movement of various fluids in a porous medium. Thus, mathematical modeling offers a real opportunity to predict and potentially optimize methods of oil and gas production, as well as treatment and prevention of groundwater pollution.

To model such filtration processes, it is necessary to select a heterogeneous porous medium. One approach here is to use continuous media with a periodic structure. The assumption of medium periodicity is a model as simply described, since it is not known how to describe real porous media. However, sometimes the results for periodic media can be generalized to more complex and stochastic continuous media. Thus, periodic media formed by a large number of cells, which can be inhomogeneous and defines a multiscale inhomogeneous porous media, will be discussed. Accordingly, the parameter characterizing the number of cells determines the small size of this cell.

Thus, it is assumed that the coefficients of the equations characterizing this model of two-phase filtration depend on two small positive parameters ε and σ. The small-scale parameter ε defines the coefficient period, which corresponds to the assumption of the periodic structure of the porous media formed by blocks and separated by high permeable faults. The value inverse to the parameter σ characterizes the relative ratio of the permeability coefficients included in the equations and corresponds to the assumption of low and high permeability of blocks and faults forming porous media. Here we will consider the case $\sigma = \varepsilon^2$. This choice is usually called the dual porosity model.

II. Homogenization for Approximation

The functions, defining the solution of this problem under consideration, model the dynamic representation of two-phase filtration (or diffusion) in porous media. But, the direct finding of these functions by computational methods is practically impossible, since for the periodic media it is necessary to choose a very fine grid, depending on ε, in order to accurately take into account the geometry of blocks and faults. In addition, the spread of the ellipticity constants of this problem will depend on σ, which will also lead to a refinement of the computational grid in order to guarantee sufficient accuracy, which requires significant resources to calculate such functions.

In such a situation, for simpler problems, the following approach was first proposed in [1, 2]. To begin with, approximate solutions to the initial problem for small ε are found. The approximate solutions usually satisfy the initial boundary value problem with constant coefficients in the domain that do not depend on ε. This initial boundary value problem with constant coefficients is usually called a homogenized or homogenized approximation to the original initial boundary value problem. Then, the homogenized problem is solved by standard computational methods. Of course, it would be desirable to have an estimate of the accuracy of this approximation. For nonlinear problems, such estimates are not always provable and one has to limit oneself to proving the appropriate convergence of these approximations. This approach has significant advantages, since solving the problem with constant coefficients is simpler and simultaneously approximates the solution to the original problem.

A detailed bibliography on the study of various aspects and homogenization methods for applying mathematical modeling to such problems in practice is given in [3, 4]. But, it is usually not taken into account that many real cellular media are formed by a large number of blocks with low permeability and separated by a connected system of faults with high permeability. Therefore, dependence on additional parameters arises when taking into account the structure of the porous medium under consideration. Here we will use the methods presented in [5, 6] for linear problems with dependence on such parameters. Following [7], we also note that filtration models typically consider homogeneous media, whose filtration coefficients and characteristics are obtained by averaging over volume without justifying this transition. However, this approach makes it unclear how to distinguish between possible filtration regimes, separating scales that are, in a sense, erased by averaging over volume.

III. Initial Boundary Value Problems

Thus, we consider nonstationary filtration processes of two-phase flows for immiscible separated liquids in porous media with a periodic structure. The Musket-Leveret model for such media is adopted as a mathematical model for describing such processes. Details on this and more general models of two-phase filtration in porous media can be found in [7, 8, 9].

This model is defined by the functions $S_w^\varepsilon, S_o^\varepsilon$, $P_w^\varepsilon, P_o^\varepsilon$, which correspond to the volume parts of the phases and the partial pressures in these phases, as a solution to the following initial boundary value problem

$$m_\varepsilon (S_o^\varepsilon)'_t - \mathrm{div}(K_\varepsilon^\sigma \Lambda_\varepsilon^o (S_o^\varepsilon)[\nabla P_o^\varepsilon - \rho_o G]) = r_\varepsilon^o G_o,$$

$$m_\varepsilon (S_w^\varepsilon)'_t - \mathrm{div}(K_\varepsilon^\sigma \Lambda_\varepsilon^w (S_w^\varepsilon)[\nabla P_w^\varepsilon - \rho_w G]) = r_\varepsilon^w G_w,$$

$$S_w^\varepsilon + S_o^\varepsilon = 1, \quad P_w^\varepsilon - P_o^\varepsilon = P_\varepsilon^c (S_w^\varepsilon).$$

This system of equations, supplemented by suitable boundary and initial conditions, is considered in the domain Ω on the time interval $(0,T)$. The indices o and w in the notations of the functions under consideration mean that these functions relate to one of the phases of liquids, for example, oil products and water. In addition, the index ε below usually means periodicity, while ε above emphasizes the dependence of the solution on this small parameter and $\sigma = \varepsilon^2$ characterizes the relative ratio of the permeability coefficients included in the equations and corresponds to the assumption of low and high permeability of blocks and faults forming porous media. In turn, the domain $\Omega = \Omega_\varepsilon^b \cup \Omega_\varepsilon^f$ is divided into two domains, which correspond to blocks and faults. It is assumed that the coefficients of the elliptic part of these equations are separated from zero, which corresponds to the absence of stagnation zones in the filtration process. By adding standard assumptions one can prove that a solution to such a problem exists, following [10], for example.

Thus, using asymptotic methods, we can pass to the limit in the problem and establish a limit homogenized problem, the solution of which is the limit functions that describe, in a certain sense, the filtration process globally. To briefly describe the homogenized problem, we note that the first terms in the equation are replaced by $m_b (S_o^b)'_t + m_f (S_o^f)'_t$ and $m_b (S_w^b)'_t + m_f (S_w^f)'_t$ with constant coefficients defined by the characteristics of the blocks and faults in natural notations. The remaining coefficients dependent on ε are also replaced by constants. The last two equations will be split into two parts for blocks and faults. But, the last equation will be determined by solving the problem on a cell for a single block. So, these will be six homogenized equations with respect to six unknowns, which are approximate solutions to the solutions of the initial problem.

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References

[1] A. Bensoussan, J. L. Lions, G. Papanicolaou, "Asymptotic analysis for periodic structures", Amsterdam, North-Holland, 1978.

[2] N. S. Bakhvalov, G. P. Panasenko, "Homogenization: averaging processes in periodic media", Kluwer, Dordrecht, 1989. doi: 10.1007/978-94-009-2247-1

[3] A. Mielke, S. Reichelt, M. Thomas, "Two-scale homogenization of nonlinear reaction-diffusion systems with slow diffusion", Netw. Heterog. Media, vol. 9:2, pp. 353-382, 2014. doi: 10.3934/nhm.2014.9.353

[4] S. Garttner, P. Knabner, N. Ray, "Local existence of strong solutions to micro-macro models for reactive transport in evolving porous media", European J. Applied Mathematics, vol. 35:1, pp. 127-154, 2024. doi: 10.1017/S095679252300013X

[5] G.V. Sandrakov, "Modeling and homogenization of filtration processes in periodic media with sources", Mathematical Modeling and Simulation of Systems. Lecture Notes in Networks and Systems, vol 1091, pp 19-32, Springer, Cham, 2024. doi: 10.1007/978-3-031-67348-1_2

[6] G.V. Sandrakov, "Asymptotic expansion methods in homogenization and some applications", A Closer Look at Homogenization. Gulcin Yildiz (ed.). Nova Science Publishers, pp. 25-130, 2024. doi: 10.52305/CIBQ6632

[7] P. Dietrich, R. Helmig, M. Sauter, H. Hotzl, J. Kongeter, G. Teutsch (eds.), "Flow and transport in fractured porous media", Springer-Verlag, Berlin, 2005. doi: 10.1007/b138453

[8] A. Ya. Bomba, V. I. Boitsov, S. V. Yaroschak, "Mathematical modeling of the process of two-phase non-isothermal filtration based on thermo-gravitational drainage technology", J. Numerical and Applied Mathematics, vol. 1 (127), pp. 5-14, 2018. (in Ukrainian).

[9] Shib Sankar Ganguli, Vijay Prasad Dimri (eds.), "Reservoir characterization, modeling, and quantitative interpretation", Elsevier, Amsterdam, 2023. isbn: 978-0-323-99593-1

[10] D. Kroener, S. J. Luckhaus, "Flow of oil and water in a porous medium", J. Differential Equations, 55:2, 276–288, 1984. doi: 10.1016/0022-0396(84)90084-6